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A simultaneous estimation procedure for the parameters of the maximum size-density line and self-thinning curve to predict stand development of fast-growing tropical species

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SHORT COMMUNICATION

A simultaneous estimation procedure for the parameters of the maximum size–density line and self-thinning curve to predict stand development of fast-growing tropical species

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Abstract A simultaneous estimation procedure for the parameters of two functions, i.e., the maximum size–density line and the self-thinning curve, is presented to predict stand development for fast-growing tropical species. This procedure assumes that the rate of periodic reduction in stand density with increasing quadratic mean diameter on a logarithmic scale (r) will increase inversely proportionally to the distance from the maximum size–density line and consequently equals the slope of the line at distance 0. Under this assumption, the maximum size–density line can be incorporated into the self-thinning curve to form an integrated equation with three parameters: k and m, the slope and constant of the maximum size–density line, and a, the rate of reduction of r of the self-thinning curve. These parameters are estimated simultaneously using measurement data on stand density, quadratic mean diameter, and the corresponding r. This procedure was evaluated by application to two data sets: 186 measurements of *Acacia mangium* and 95 measurements of *Paraserianthes falcataria*, for which the parameters k and m have previously been calculated. The parameters estimated using this procedure were in good agreement with previous ones based on the *A. mangium* data set, and the differences found for the *P. falcataria* data set were also small, within the error variances. Therefore, it is concluded that the proposed procedure would give almost the same estimates from a single calculation step as the previous procedure that required two separate calculation steps.

Keywords *Acacia mangium* · Maximum size–density line · *Paraserianthes falcataria* · Quadratic mean diameter · Self-thinning curve

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Introduction

An intentional shift toward solid timber production has become evident in the planting of fast-growing tropical species, not only *Paraserianthes falcataria* but also *Acacia mangium* and *Eucalyptus*, which have been planted to supply raw material for pulp and paper (Cossalter and Pye-Smith 2003). This trend may be primarily due to the higher profitability of solid timber production, but is also due to increasing demand for substitutes for reduced timber supply from natural forests. Therefore, the role of stand density control in plantations has become more important than ever before to ensure sustainable supply of target size of logs to meet diversified production purposes, such as furniture, finger-joining wood, and veneer.

Stand density control, aiming to optimize tending regime in the plantation, requires a growth model able to predict stand development based on the size–density relationship. This type of growth model, although it has been widely used in Japan (Yoda et al. 1963; Tadaki 1964; Ando 1968), is still limited for fast-growing tropical species. However, a few of them have been applied for *A. mangium* and *P. falcataria*, using the relationship between stand density and quadratic mean diameter (Kurinobu et al.

2006, 2007); first the maximum size–density line was determined, then the self-thinning curve was derived. The size–density relationship has been recognized as variable even within a species, because of the controversy regarding the generality of the 3/2 power low (Weller 1987; Zeide 1987; Osawa and Sugita 1989; Sackville Hamilton et al. 1995). Therefore, in practice, a simplified procedure for estimating the parameters of the maximum size–density line and the self-thinning curve is desirable, so that the model can be easily revised for target environments for plantation establishment.

In this paper, a simultaneous estimation procedure for the parameters of the maximum size–density line and self-thinning curve was derived by assuming that the rate of periodic reduction in stand density with increasing quadratic mean diameter on a logarithmic scale will increase inversely proportionally to the distance from the maximum size–density line and consequently equals the slope of the line at distance 0. Then, the procedure is examined by application to *A. mangium* and *P. falcataria* data sets whose parameters were previously calculated (Arisman et al. 2004; Kurinobu et al. 2008).

Materials and method

The maximum size–density line for quadratic mean diameter (d_q) is expressed with the following linear equation on a log–log scale of stand density [$N = \log(n)$, n: number of stems per ha] and diameter [$D = \log(d_q)$] (Reineke 1933):

$$N = m + kD, \tag{1}$$

where k and m are the slope of the line and a constant, respectively. The distance between the line and the measurement (L) is given by the following formula (Arisman et al. 2004):

$$L = xy/(x^2 + y^2)^{1/2}, \tag{2}$$

where x is the distance between the measurement (D, N) and the size–density line in the horizontal direction: $x = (N - m)/k - D$, and y is the distance in the vertical direction: $y = m + k\cdot D - N$. Thus the relationship between x and y is expressed as $y = -k\cdot x$, and replacing y in Eq. 2 with this relationship gives the following equation:

$$L = -kx/(1 + k^2)^{1/2}. \tag{3}$$

Mortality caused by self-thinning is expressed as a rate of periodic reduction in stand density (ΔN) with increasing quadratic mean diameter (ΔD) on a logarithmic scale (Kurinobu et al. 2006). The trend for the rate ($r = \Delta N/\Delta D$) as a function of distance (L) was fitted with an exponential function (Eq. 4), instead of the Mitscherlich function used in the previous study. This is because the simple exponential function can satisfy the assumption that the rate will equal the slope of the line at distance 0, although both functions can express the general trend that the rate will increase inversely proportionally to the distance from the maximum size–density line.

$$r = k\exp(aL), \tag{4}$$

where a is the rate of reduction of r in the self-thinning curve. Equation 4 can be rearranged by replacing L with Eq. 3 and x with $(N - m)/k - D$. In this way, the maximum size–density line is incorporated into the self-thinning curve, leaving parameters k and m, as shown in Eq. 5.

$$r = k\exp\left[a(m + kD - N)/\left(1 + k^2\right)^{1/2}\right]. \tag{5}$$

The three parameters in Eq. 5 (a, k, and m) can be estimated simultaneously using measurement data on stand density (N), quadratic mean diameter (D), and the corresponding r ($=\Delta N/\Delta D$) by using the Gauss–Newton iteration method with a set of partial derivatives of the equation with respect to the parameters (Snedecor and Cochran 1989; see Appendix online).

This procedure was evaluated using two data sets: 283 diameter measurements from 51 permanent plots of *A. mangium*, and 128 measurements from 32 plots of *P. falcataria* (Table 1). The permanent plots were 0.1 ha in size for both species, and diameter at breast height (DBH) was measured for all surviving trees. Periodic measurement, twice per year, was conducted for 3 years for *P. falcataria*, while the measurement for *A. mangium* was repeated almost annually until 8–9 years old. The parameters of size–density lines of both data sets, k and m, were previously calculated by minimizing the average distance between measurements and the maximum size–density line, i.e., the minimum distance boundary method (Arisman et al. 2004; Kurinobu et al. 2008).

The periodic reduction in stand density (ΔN) on a logarithmic scale and the increment of quadratic mean diameter (ΔD) were calculated using 183 pairs of consecutive

Table 1 *A. mangium* and *P. falcataria* data sets for application of the simultaneous estimation procedure

Species/trait	*A. mangium* Mean	*P. falcataria* Mean
Number of plots	51 (283)	32 (128)
Age (years)	5.9 (3–10)	5.4 (3–8)
DBH (cm)	14.5 (6.4–23.1)	18.0 (10.2–31.4)
Stem density (ha^{-1})	1,168 (540–2,210)	878 (300–1,570)

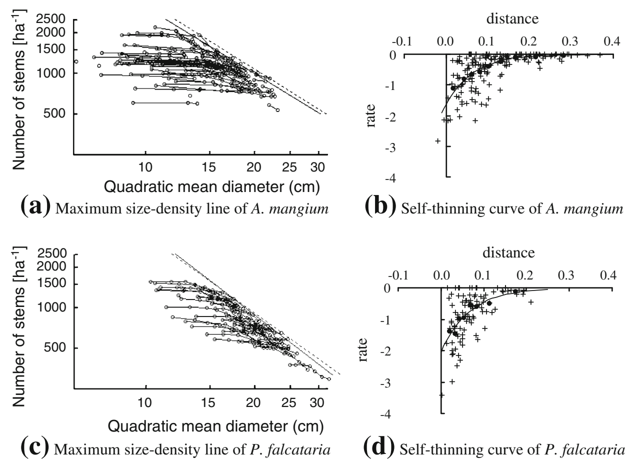
The number in parenthesis associated with the number of plots is the total number of measurements, and those for age, DBH, and stem density are measurement ranges measurements of *A. mangium* and 95 pairs for *P. falcataria*, respectively. Then, the simultaneous estimation procedure was applied to these pairs of data, i.e., the rate $[r = \Delta N/\Delta D = (N_{t+1} - N_t)/(D_{t+1} - D_t)]$ and the corresponding stand density (N_t) and quadratic mean diameter (D_t), to estimate the parameters *a*, *k*, and *m*, as shown in Eq. 5. In the *P. falcataria* data set, there was a measurement with abrupt reduction in stand density due to illegal logging by encroachment, which was thus omitted from the analysis.

Results and discussion

The three parameters estimated by the simultaneous estimation procedure for the two data sets are presented together with the previous estimates of *k* and *m* in Table 2. For *A. mangium*, the parameters estimated using this procedure agreed well with the previous ones. The differences found for *P. falcataria* were also small, within the error variances. These results suggest that the proposed procedure would give almost the same estimates as the previous method. However, the coefficients of determination were generally lower, being 0.472 for *A. mangium* and 0.335 for *P. falcataria*, due to instability of the rate, which varies from plot to plot depending on conditions.

These results were further examined by plotting the respective maximum size–density line and self-thinning curve for the two species (Fig. 1). There were a few measurements slightly above the maximum size–density line estimated by the proposed procedure for both species (Fig. 1a, c). These results may be regarded as a possible consequence of the parameter estimation, because this procedure simply determines the three parameters to maximize the correlation between the rate and the estimates according to Eq. 5. The self-thinning curve showed fairly good agreement with the average rates for the eight groups of measurements that were sorted by their distances (Fig. 1b, d). Thus, the proposed procedure will capture a trend for the rate reasonably well on an average basis even with large plot-to-plot variation.

The assumption for the trend of the rate ($\Delta N/\Delta D$), which becomes steeper as it gets closer to the maximum size–density line and consequently equals the slope of the line at distance 0, is known as density-dependent mortality (Tadaki 1963; Osawa and Sugita 1989). Unlike in other studies, this trend was accounted for by the distance between measurements and the maximum size–density line, and then the distance itself was absorbed, yielding a single equation (Eq. 5) with three unknown parameters: *a*, *k*, and *m*, for simultaneous solution. Therefore, the procedure presented herein is regarded as one of the mathematically integrated solutions to account for density-related



(a) Maximum size-density line of *A. mangium* **(b)** Self-thinning curve of *A. mangium*
(c) Maximum size-density line of *P. falcataria* **(d)** Self-thinning curve of *P. falcataria*

Fig. 1 Maximum size–density line and self-thinning curve plotted using the parameters resulting from the simultaneous estimation procedure (Table 2). **a**, **c** Agreement of maximum size–density line for *A. mangium* and *P. falcataria*; *solid line* simultaneous estimation procedure, *dashed line* minimum distance boundary method. **b**, **d** Trend between the rate ($r = \Delta N/\Delta D$) and the distance between the measurement and the maximum size–density line. *Closed circles* are the average rates for the eight groups of measurements that were sorted by their distances, and the estimated trend is shown by the *solid line*

Table 2 Parameters estimated by the simultaneous estimation procedure and by the minimum distance boundary method for the *A. mangium* and *P. falcataria* data sets

Parameters	*A. mangium*		*P. falcataria*	
	Simultaneous estimation procedure	Minimum distance boundary method	Simultaneous estimation procedure	Minimum distance boundary method
a	−15.397 (1.698)		−15.230 (2.920)	
k	−1.638 (0.109)	−1.67	−2.060 (0.138)	−1.912
m	5.129 (0.138)	5.204	5.625 (0.193)	5.445
R^2	0.472		0.335	

Parameters *a*, *k*, and *m* are shown in Eq. 5. *k* and *m* by minimum distance boundary method were from Arisman et al. (2004) for *A. mangium* and Kurinobu et al. (2008) for *P. falcataria*. Standard errors for parameter *a*, *k*, and *m* are given in parenthesis. R^2 is the coefficient of determination mortality, focusing on the relationship between quadratic mean diameter and stand density.

Density-related mortality was well described with this procedure on an average basis, as shown in Fig. 1b, d. However, the coefficients of determination were generally low, due to the nature of the rate, which is prone to be affected by plot-to-plot variation in periodic growth and mortality (Table 2). Thus, this procedure requires either a data set with sufficient number of measurements to offset plot-to-plot variation of the rate or proper stratification of plots by site index classes, etc. to reduce the variation in order to estimate parameters suitable for growth modeling work.

A practical advantage of this procedure is its simplicity, estimating the three parameters a, k, and m with a single calculation step using the rate as a response variable and D and N as independent variables in Eq. 5. Even with this simplified procedure, the estimated parameters showed fairly good agreement with previous results (Table 2). Therefore, this procedure would be a convenient tool to meet the immediate need for developing growth models as well as for their repeated update based on rapidly accumulating data, as is usually the case with fast-growing tropical species.

Another desirable property of this procedure is that it is free from data editing when determining the maximum size–density line, because it utilizes all data for the rate and corresponding D and N values to estimate the three parameters in Eq. 5. On the contrary, the previous procedures, such as principal component analysis or reduced major axis (RMA) regression (Sackville Hamilton et al. 1995), both require data sets from overcrowded stands selected from hundreds of measurements (Osawa and Sugita 1989). The minimum distance boundary method may alleviate this problem, but it may be biased by data with measurement error due to its assumption that all measurements are below the line. Hence, this procedure has good potential to provide more consistent estimates of the maximum size–density line as long as the data set contains the rates of overcrowded stands.

In conclusion, a simultaneous estimation procedure for the parameters of two functions, i.e., the maximum size–density line and the self-thinning curve, was derived to account for density-related mortality. The parameters estimated using this procedure agreed well with those for the maximum size–density line derived by previous methods, and also described the average trend of self-thinning reasonably well. Therefore, due to its simplicity, this procedure would be suitable for use in growth modeling work for fast-growing tropical species where frequent update is anticipated.

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